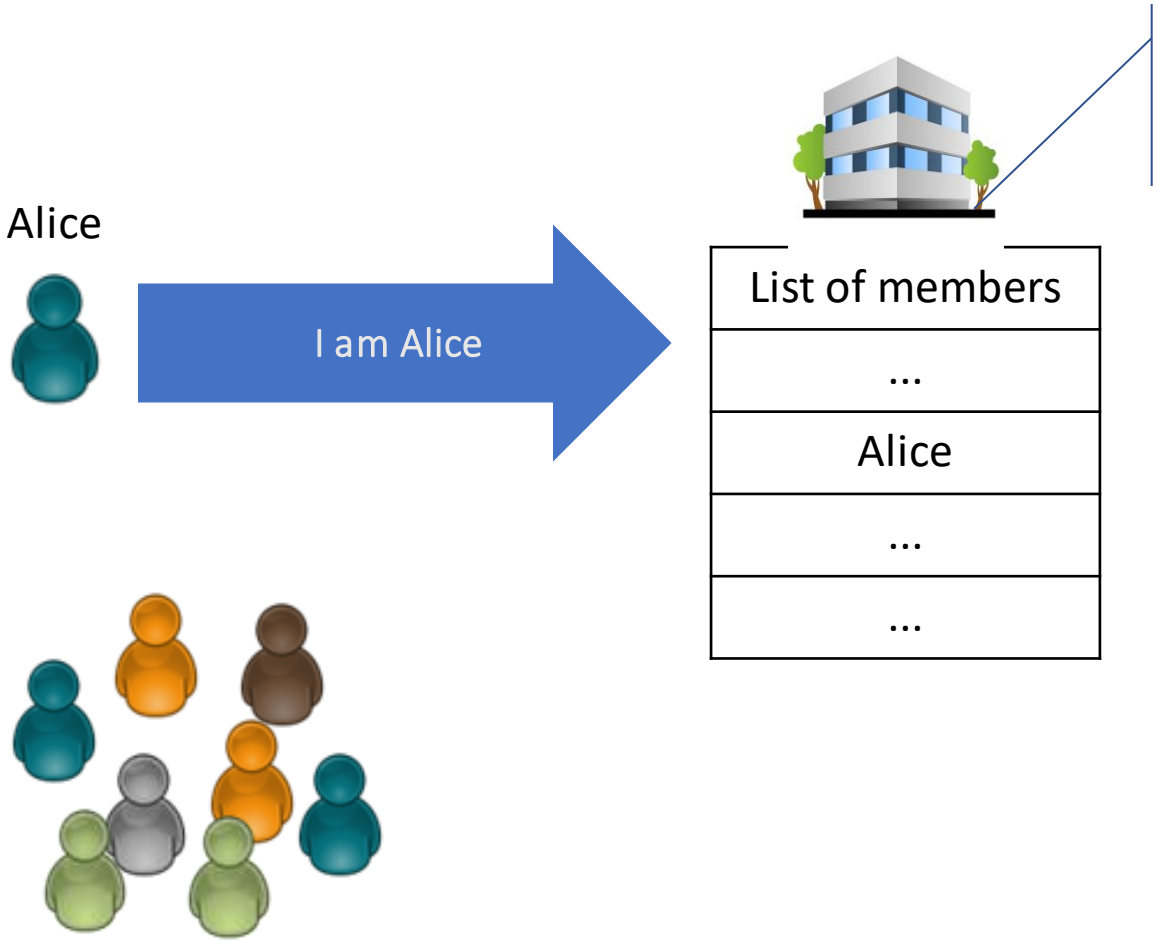
Efficient Constructions of Bilinear Accumulators

Ioanna Karantaidou, Foteini Baldimtsi

GEORGE MASON UNIVERSITY

Set Membership



List of members as a Data structure

- **Size** of List: O(n)
- (at least one of) **Additions/Deletions,** lookups depends on n
- **Privacy** against list holder/membership verification in a privacy preserving way: Expensive!

Accumulator Setting

MANAGER

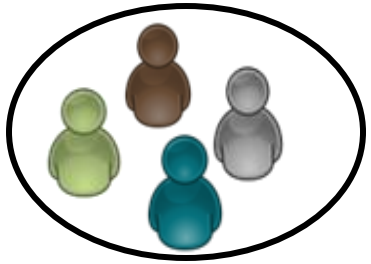
Initialize & Create Acc.v

Acc.v

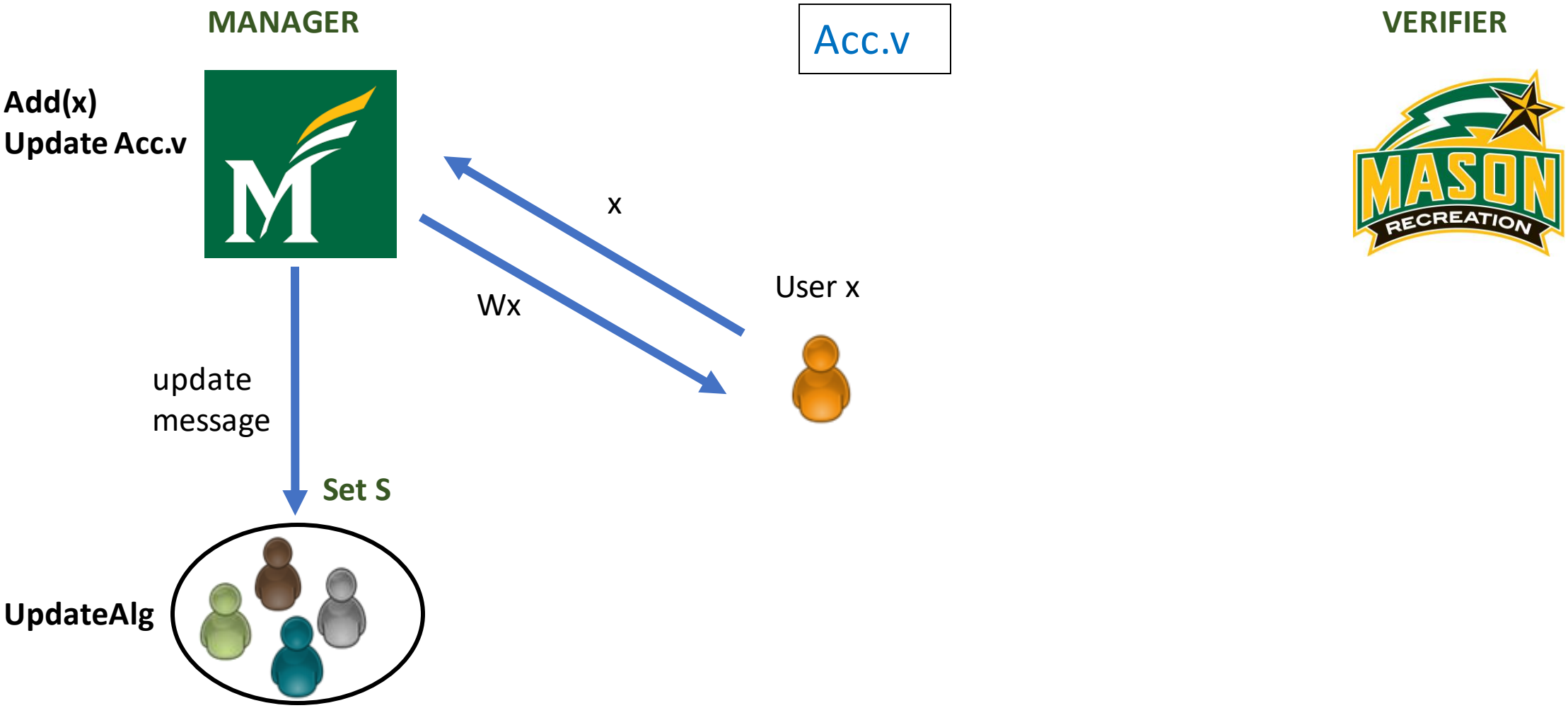
Accumulator Value: holds Set S

VERIFIER

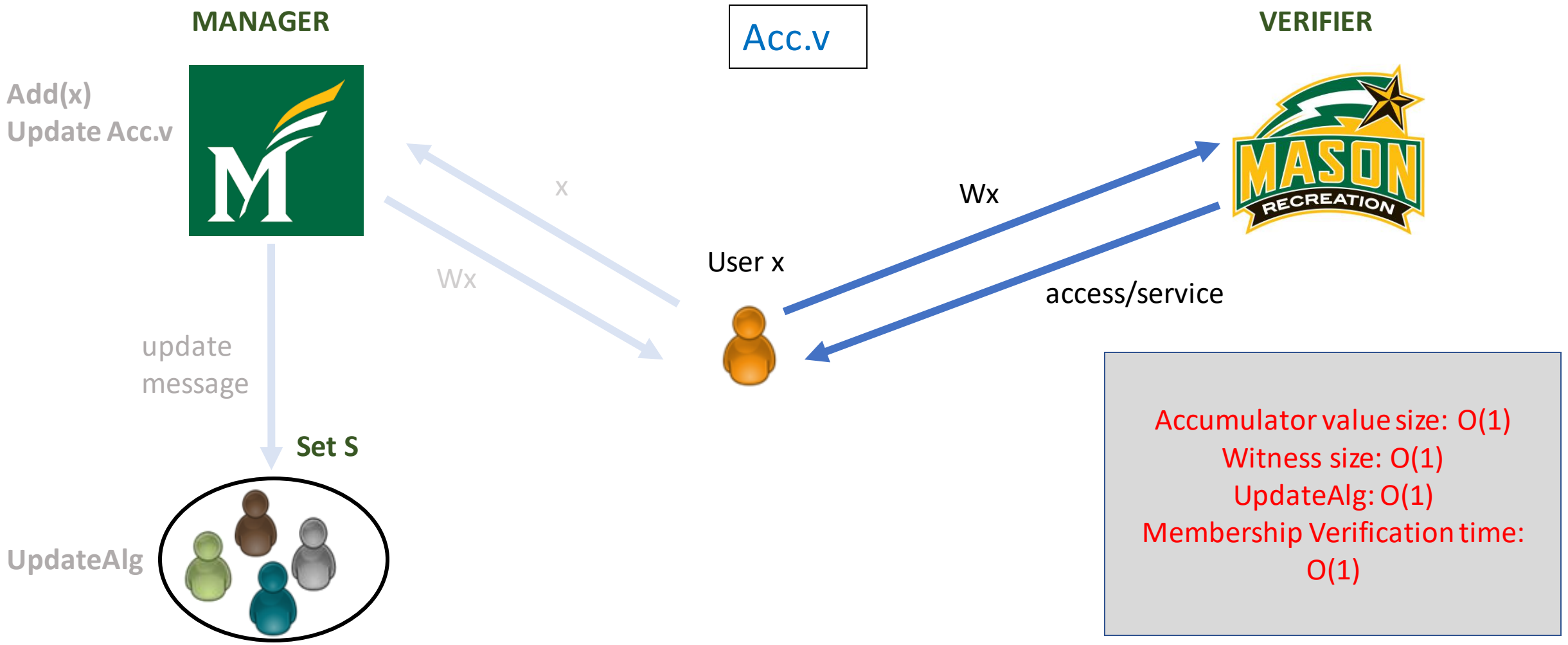
Set S



Positive Accumulator: adding User x



Positive Accumulator: proving membership



Security Properties (membership)

...
Alice
...

Set/List *Verification=lookup*	Accumulator acc *Verification algorithm:VerMem(w_x)*
Alice is a member → verification ✓	$x \in acc$ → VerMem(w_x)=1

correctness

Charlie
Alice

Bob is **not** a member → verification ✗	$x \notin acc$ → VerMem(w_x)=0 (or =1 with negligible prob.)

soundness

2 Types of Accumulators

RSA based accumulators [CL02, LLX07, BdM93]

- Accumulate odd prime numbers
- Factorization of group hidden
- Strong RSA assumption

Bilinear Pairing based accumulators [N05, CKS09, ATSM09, ZKP17]

- Accumulate integers
- Known order groups
- Witness, accumulator value belong in pairing friendly groups
- q-SDH assumption

Choice depends on the application!

Common Issues with Known Accumulators

- Unnecessary accumulator updates that cause high communication costs
- Expensive non-membership operations
- Computational overhead due to extra properties

Can we do better if we take advantage of the presence of a trusted entity (manager)?

Discussion on the secret key model

- Most known constructions have a trusted setup
- Anonymous Credentials, subscription-based services, etc

Our Results

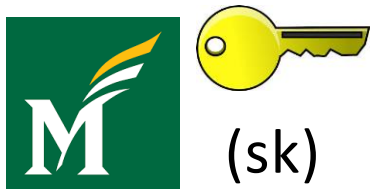
1. Positive Bilinear Accumulator with Optimal Communication Cost
2. Universal Bilinear Accumulator with Constant Non-Membership Witness Creation
3. ZK Accumulator with Constant Non-Membership Witness Creation and Update

FIRST CONSTRUCTION

Positive Bilinear Accumulator with Optimal Communication Cost

Positive Bilinear Accumulator

$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)}$$



(sk)

Add(x)

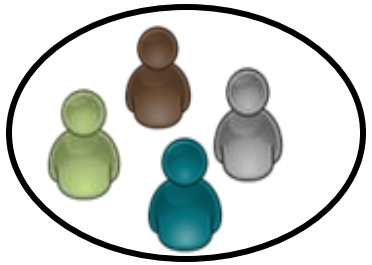
User x

$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)(x+sk)}$$

upmsg

$$w_x = g^{(x_1+sk)\ldots(x_n+sk)}$$

Set S



Positive Bilinear Accumulator Verification

$$w_x = Acc.v^{(x+sk)^{-1}}$$

$$w_x{}^{(x+sk)} = Acc.v$$

Public parameters: $g, g^{sk}, (g^{sk})^2, (g^{sk})^3, \ldots \rightarrow$

$w_x{}^{(x+sk)}$ ✗

$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)(x+sk)}$$

$$w_x = g^{(x_1+sk)\ldots(x_n+sk)}$$

Public parameters:

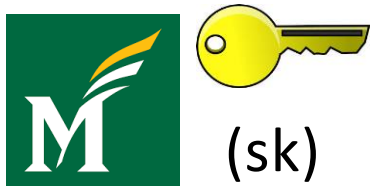
$g, g^{sk}, (g^{sk})^2, (g^{sk})^3, \ldots \rightarrow$

g^x, g^{sk}

e(w_x, g^x g^{sk})=e($Acc.v$, g) (VerMem)

Positive Bilinear Accumulator

$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)(x+sk)}$$



(sk)

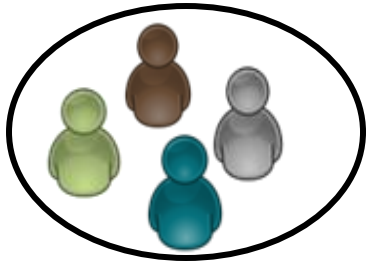
Del(x)

User x

upmsg

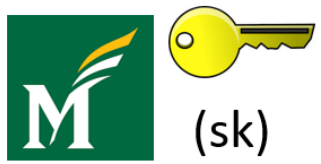
$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)}$$

Set S



Positive Bilinear Accumulator

$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)}$$



(sk)

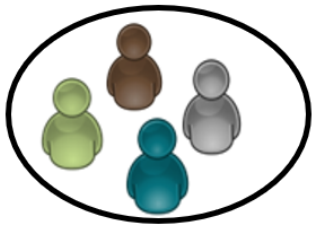
Add(x)

User x

$$Acc.v = g^{(x_1+sk)\ldots(x_n+sk)(x+sk)}$$

upmsg

Set S

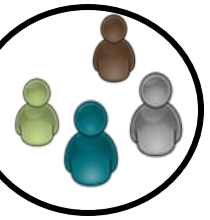


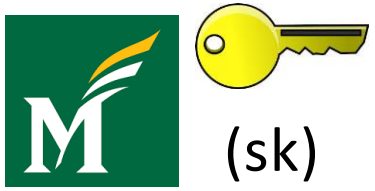
$$w_x = g^{(x_1+sk)\ldots(x_n+sk)}$$

Minimum communication bound (on update messages) for positive accumulators= |d| (number of deletions)

Camacho, Philippe, and Alejandro Hevia. "On the impossibility of batch update for cryptographic accumulators." *International Conference on Cryptology and Information Security in Latin America*. Springer, Berlin, Heidelberg, 2010.

Positive Bilinear Accumulator with Optimal Communication Cost-First try





(sk)

User x

$$Acc.v = g^{u}$$

Add(x)

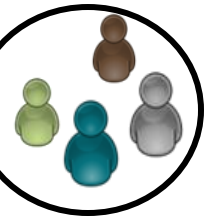
$$w_x = g^{u\,(x+sk)^{-1}}$$

Del(x)

upmsg

$$Acc.v = g^{u\,(x+sk)^{-1}}$$

Positive Bilinear Accumulator with Optimal Communication Cost-First try



(sk)

User x

- Communication efficient
- Dynamic (add,del)
- Positive (membership)

- Correctness

$w_x^{(x+sk)} = Acc.v$

holds and **VerMem** same

- Soundness??

Acc

Positive Bilinear Accumulator with Optimal Communication Cost-First try

Proof overview:

- R (public parameters) runs an adversary A (public parameters)
- A submits lists of to-be-added, to-be-deleted elements L_A, L_D
- R simulates updates and witnesses
- A breaks acc soundness
- R breaks q-SDH assumption

q-SDH: Given (p, G, G_T, e, g), $\{g^{sk}\}^i, i = 0, \dots, q$ there is negligible probability of finding

$g^{\frac{1}{sk+x}}$ for $x \in \mathbb{Z}_p$

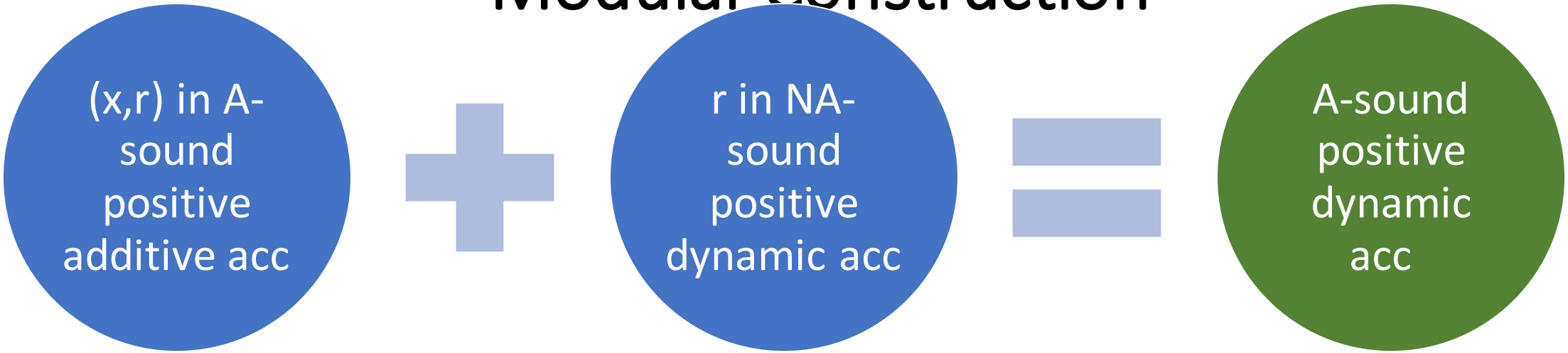
Positive Bilinear Accumulator with Optimal Communication Cost-First try

Proof overview:

- R (public parameters) runs an adversary A (public parameters)
- A submits lists of to-be-added, to-be-deleted elements L_A, L_D
- R simulates updates and witnesses
- A breaks acc soundness
- R breaks q-SDH assumption

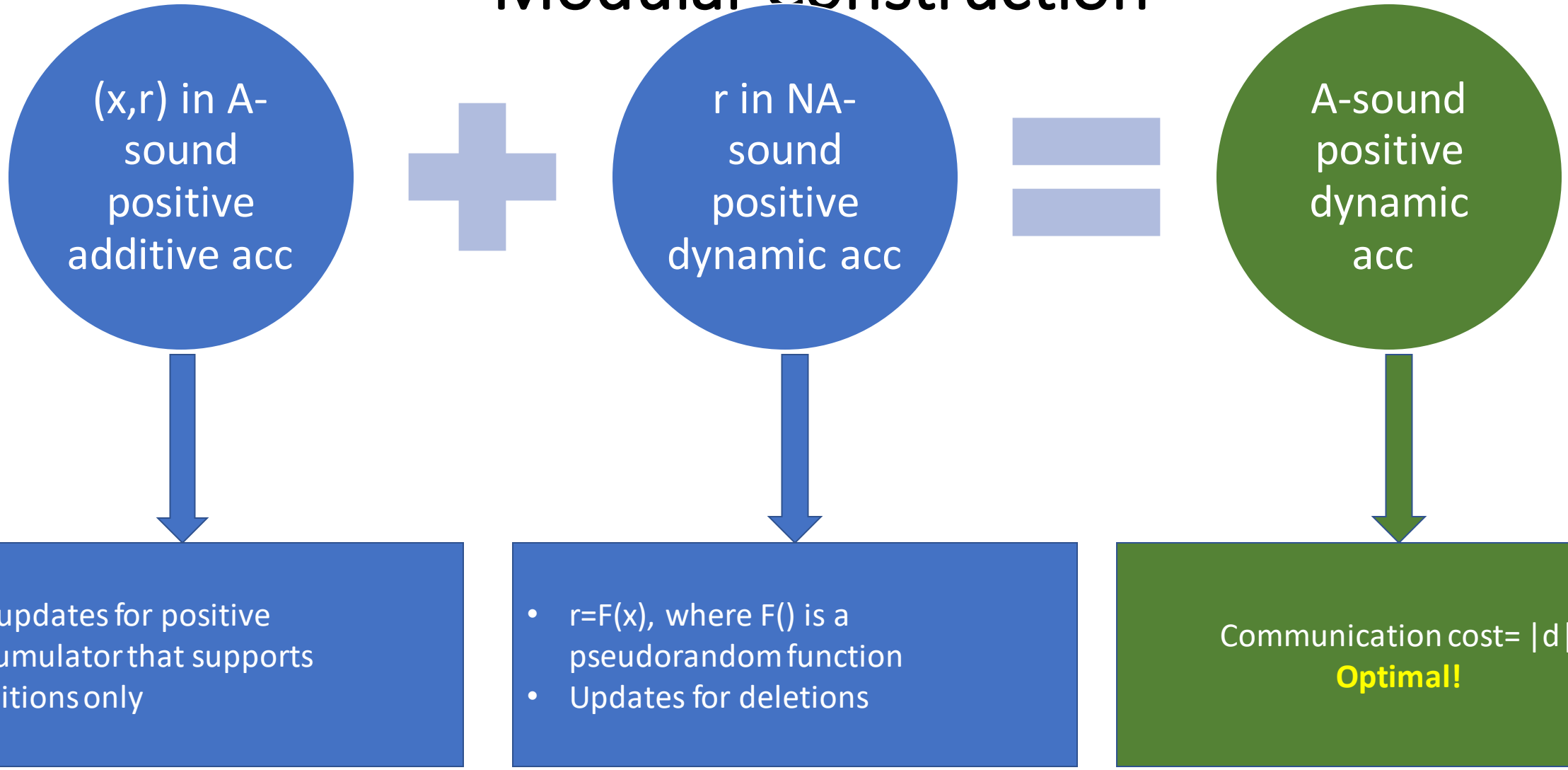
Adaptive soundness not achieved

Positive Bilinear Accumulator with Optimal Communication Cost-Modular Construction



Baldimtsi, Foteini, et al. "Accumulators with applications to anonymity-preserving revocation." *2017 IEEE European Symposium on Security and Privacy (EuroS&P)*. IEEE, 2017.

Positive Bilinear Accumulator with Optimal Communication Cost- Modular Construction



	Positive			
	Camenisch et al 09	Nguyen 05	this work (NA-sound)	this work (A-sound)
Add	1	1	1	1
Del	1	1	1	1
MemWitCreate	1	1	1	1
NonMemWitCreate	-	-	-	-
MemWitUpOnAdd	1	1	0	0
MemWitUpOnDel	1	1	1	1
NonMemWitUpOnAdd	-	-	-	-
NonMemWitUpOnDel	-	-	-	-
VerMem	1	1	1	1
VerNonMem	-	-	-	-
Manager storage	1	1	1	1
Parameters	2q	q	q	q
Com. cost	|a|+|d|	|a|+|d|	|d|	|d|
Efficient ZKPs	✓	✓	✓	✓
Adaptively-sound	✓	✓		✓

- Jan Camenisch, Markulf Kohlweiss, and Claudio Soriente. An accumulator based on bilinear maps and efficient revocation for anonymous credentials. In PKC 2009
- Lan Nguyen. Accumulators from bilinear pairings and applications. In CT-RSA 2005.

SECOND CONSTRUCTION

Universal Bilinear Accumulator with Constant Non-Membership Witness Creation

Additional Properties (non-membership:NM)

Charlie
Alice

Set/List

NM verification=lookup

Accumulator acc

NM verification algorithm:
VerNonMem($\overline{w_x}$)

Bob is **not** a member
→ NM verification ✓

$x \notin acc$ → VerNonMem($\overline{w_x}$)=1

correctness

...
Alice
...

Alice is a member
→ NM verification ✗

$x \in acc$ → VerNonMem($\overline{w_x}$)=0
(or =1 with negligible prob.)

soundness

Generic Universal Modular Construction motivation: Non membership for y

Bilinear **ATSM09**, S=$\{x_i\}$, $x_i \in \mathbb{Z}_p$

$$\prod_{i=1}^{|S|}(x_i+sk) = c(sk)(y+sk)+d$$

Users (public parameters):

S=$\{x_i\}$, polynomial division

Manager (sk):

$\prod_{i=1}^{|S|}(x_i+sk) \in \mathbb{Z}$, used as exponent

RSA **LLX07**, S=$\{y_i\}$, y_i primes

a $(\prod_{i=1}^{|S|} y_i) + b\ y = 1$

Users (public parameters)/Manager (sk):

$\prod_{i=1}^{|S|} y_i \in \mathbb{Z}$, Euclidean algorithm

Generic Universal Modular Construction motivation: Non membership for y

Bilinear **ATSM09**, S=$\{x_i\}$, $x_i \in \mathbb{Z}_p$

$$\prod_{i=1}^{|S|}(x_i+sk) = c(sk)(y$$

Users (public paran
S=$\{x_i\}$, polynomial
Manager (sk).

$\prod_{i=1}^{|S|}(x_i+sk) \in \mathbb{Z}$, used as exponent

non-membership cost: |S|

RSA **LLX07**, S=$\{y_i\}$, y_i primes

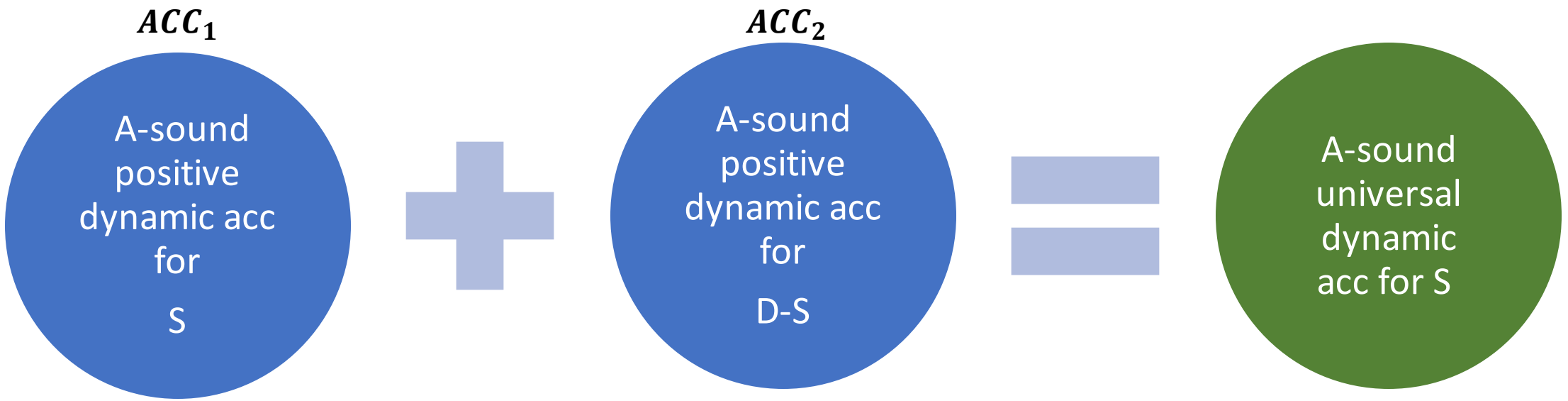
$\prod_{i=1}^{|S|} y_i) + b\ y = 1$
blic parameters)/Manager (sk):

$\prod_{i=1} y_i \in \mathbb{Z}$, Euclidean algorithm

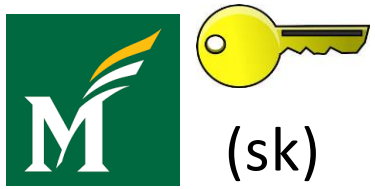
Generic Universal Modular Construction Overview

Can we replace non-membership with constant-runtime membership??
Yes, with a trusted manager

Can we make sure that ACC_1 and ACC_2 are disjoint?
The accumulator manager always signs the most up to date value of the accumulator



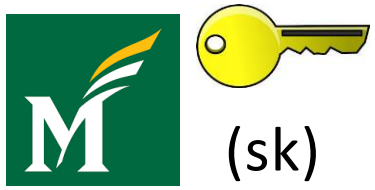
Generic Universal Modular Construction



(sk)

$\boldsymbol{ACC_1.\,Gen(1^{\lambda}, \emptyset)}$
$\boldsymbol{ACC_2.\,Gen(1^{\lambda}, D)}$

Generic Universal Modular Construction



(sk)

User x

$x \in S_2$

$ACC_1(S_1)$ $ACC_2(S_2)$

Add(x)

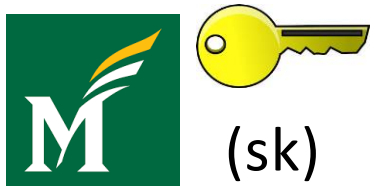
$ACC_1.\mathbf{add}(\mathbf{x})$

$ACC_2.\mathbf{del}(\mathbf{x})$

$ACC_1(S_1 \cup \{x\})$ $ACC_2(S_2 - \{x\})$

$w_x = ACC_1.w$

Generic Universal Modular Construction



(sk)

User x

$x \in S_1$

$ACC_1(S_1)$ $ACC_2(S_2)$

Del(x)

$ACC_1.\mathbf{del}(\mathbf{x})$
$ACC_2.\mathbf{add}(\mathbf{x})$

$ACC_1(S_1 - \{x\})$ $ACC_2(S_2 \cup \{x\})$

$\overline{w_x} = ACC_2.w$

w $(membership)$
$\overline{w}$ (non-membership)

Generic Universal Modular Construction

User x

VERIFIER

w_x

$\boldsymbol{ACC_1.\mathrm{VerMem}(w_x)}$

User y

O(1)

$\overline{w_y}$

$\boldsymbol{ACC_2.\mathrm{VerMem}(\overline{w_y})}$

Generic Universal Modular Construction

- **Gen**(1^λ)**:**
  1. $\mathsf{ACC}_1.Gen(1^\lambda, \emptyset)$
  2. $\mathsf{ACC}_2.Gen(1^\lambda, D)$ with the same random coins for parameters, where D is the accumulator's domain
- **Add**(x)**:**
  1. $\mathrm{ACC}_1.\mathrm{Add}(x)$
  2. $w_x = \mathsf{ACC}_1.\mathrm{MemWitCreate}(x)$
  3. $\mathsf{ACC}_2.\mathrm{Del}(x)$
- **Del**(x)**:**
  1. $\mathsf{ACC}_2.\mathrm{Add}(x)$
  2. $\overline{w_x}$=$\mathsf{ACC}_2.\mathrm{MemWitCreate}(x)$
  3. $\mathsf{ACC}_1.\mathrm{Del}(x)$
- **MemWitUpOnAdd/Del(y):**
  1. ACC_1.MemWitUpOnAdd/Del(y)
- **NonMemWitUpOnAdd/Del (y):**
  1. ACC_2.MemWitUpOnAdd/Del(y)
- **VerMem**(x)**:**
  1. $\mathsf{ACC}_1.\mathrm{VerMem}(x)$
- **VerNonMem**(x)**:**
  1. $\mathsf{ACC}_2.\mathrm{VerMem}(x)$

Fig. 3. Generic Construction for Universal Dynamic accumulator

Note on Efficiency

Concretes:

- Generation (run once) linear to Domain size
- Add/Del of double cost

Asymptotics:

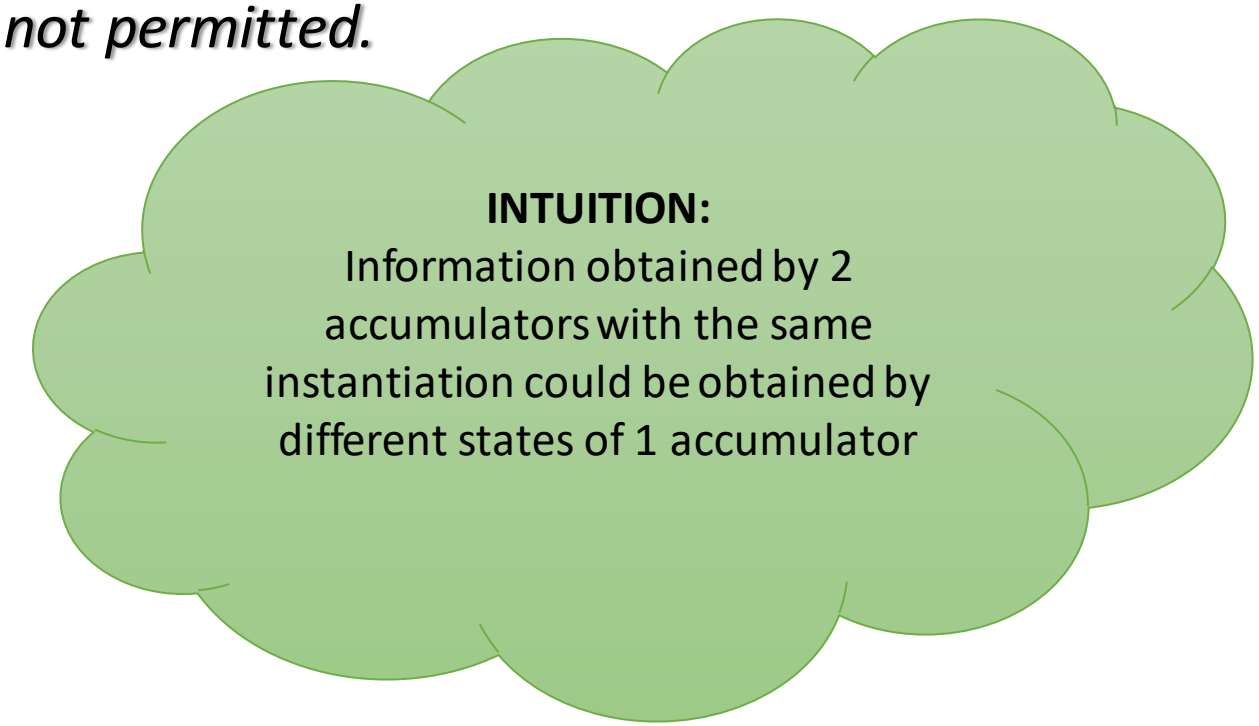
All operations constant, independent of accumulated set S

Generic Universal Modular Construction- Soundness

Theorem: *A combination of accumulators* ACC_1, ACC_2 *is a universal dynamic adaptively-sound accumulator if* ACC_1, ACC_2 *are positive dynamic adaptively-sound accumulators of domain D and one is holding* $S \subset D$ *and the other one* $\overline{S} \subset D$ *and public updates are not permitted.*

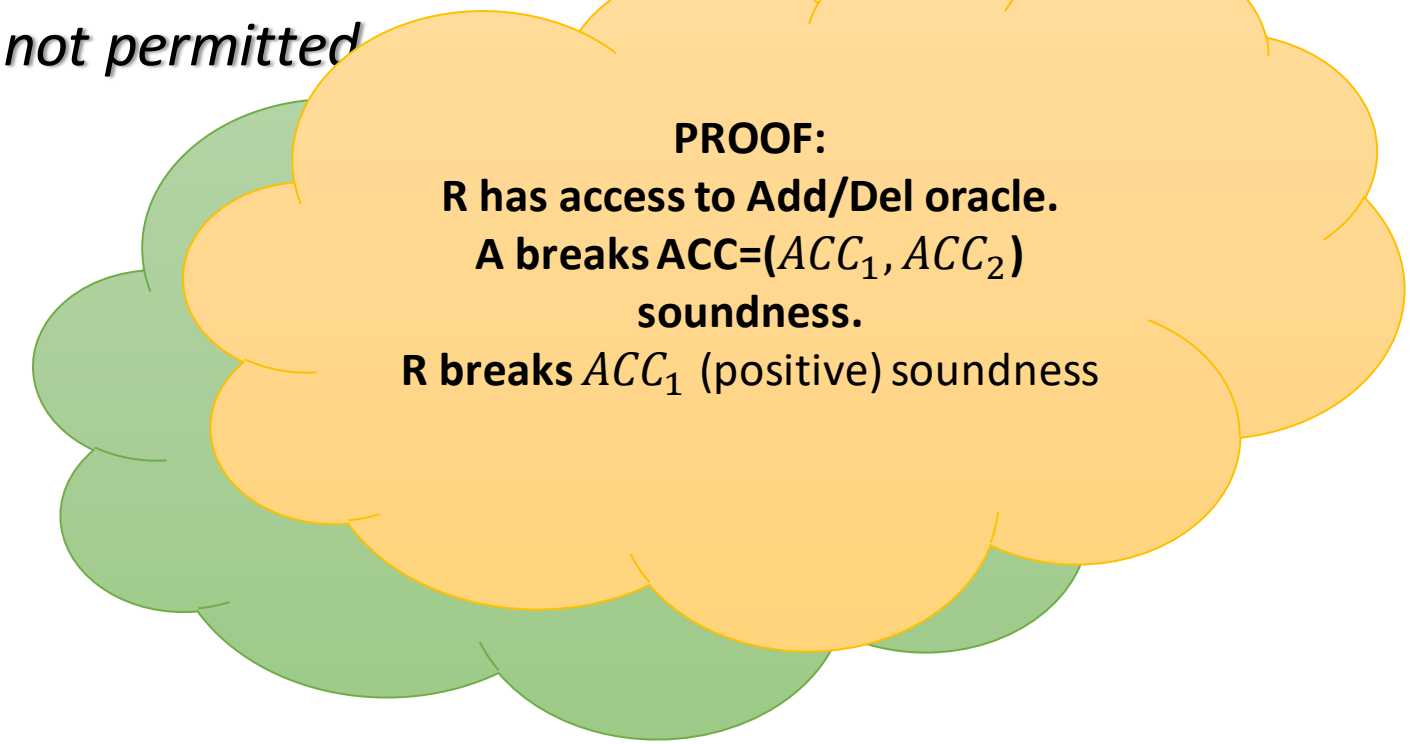
Generic Universal Modular Construction Soundness

Theorem: *A combination of accumulators* ACC_1, ACC_2 *is a universal dynamic adaptively-sound accumulator if* ACC_1, ACC_2 *are positive dynamic adaptively-sound accumulators of domain D and one is holding* $S \subset D$ *and the other one* $\overline{S} \subset D$ *and public updates are not permitted.*



Generic Universal Modular Construction Soundness

Theorem: *A combination of accumulators* ACC_1, ACC_2 *is a universal dynamic adaptively-sound accumulator if* ACC_1, ACC_2 *are positive dynamic adaptively-sound accumulators of domain D and one is holding* $S \subset D$ *and the other one* $\overline{S} \subset D$ *and public updates are not permitted*



Efficiency Results

	Positive			Universal	
	Camenisch et al 09	Nguyen 05	this work (A-sound)	Au et al 09	This work- Instantiation with Nguyen 05
Add	1	1	1	1	1
Del	1	1	1	1	1
MemWitCreate	1	1	1	1	1
NonMemWitCreate	-	-	-	|S|	1
MemWitUpOnAdd	1	1	0	1	1
MemWitUpOnDel	1	1	1	1	1
NonMemWitUpOnAdd	-	-	-	1	1
NonMemWitUpOnDel	-	-	-	1	1
VerMem	1	1	1	1	1
VerNonMem	-	-	-	1	1
Manager storage	1	1	1	|S|	1
Parameters	2q	q	q	q	q?
Com. cost	|a|+|d|	|a|+|d|	|d|	|a|+|d|	|a|+|d|
Efficient ZKPs	✓	✓	✓	✓	✓
Adaptively-sound	✓	✓	✓	✓	✓

THIRD CONSTRUCTION

ZK Accumulator with
Constant Non-Membership Witness Creation and Update

Application: ZK Accumulator

Acc.v

MANAGER

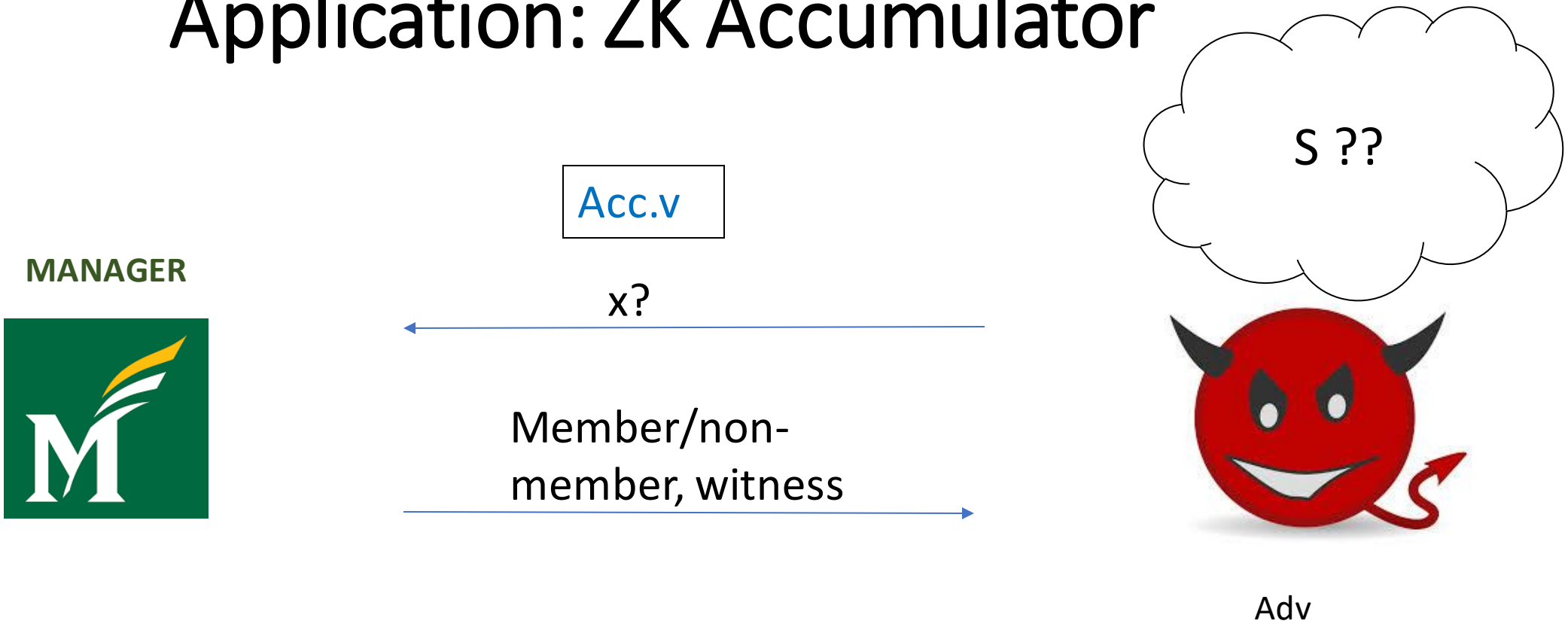
x?

Member/non-member, witness

Adv

Esha Ghosh , Olga Ohrimenko , Dimitrios Papadopoulos , Roberto Tamassia and Nikos Triandopoulos "Zero-Knowledge Accumulators and Set Operations" *IACR Cryptology ePrint Archive* 2015 (2015): 404.

Application: ZK Accumulator



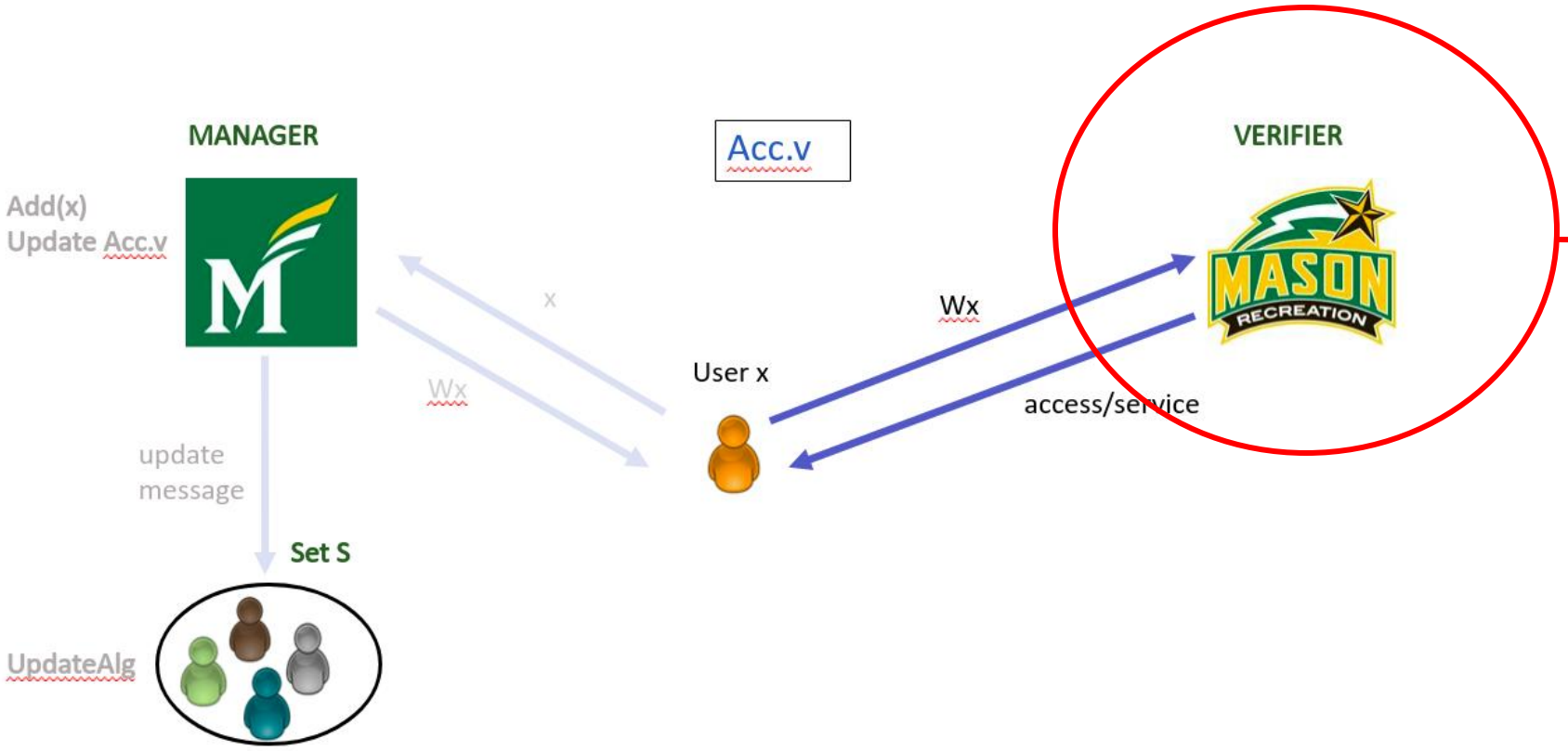
Goal:

- Adv can learn only the latest query answers

How:

- Randomness in the exponent

Application: ZK Accumulator- Why



1. Verifier is ZK adversary
2. ZK accumulator is managed by a trusted entity
3. Randomness harms efficiency

Application: ZK Accumulator- the need for randomness

1. First element's witness is g (generator is public information)
2. A guess about S can be verified with public information
3. A witness can be updated with public information (still valid?)

Public parameters: $g, g^{sk}, (g^{sk})^2, (g^{sk})^3, \ldots$

$$S = \{x_1, x_2\}$$

$$acc.v = g^{(x_1+sk)(x_2+sk)} = g^{x_1x_2+(x_1+x_2)sk+sk^2} = g^{x_1x_2}(g^{sk})^{x_1+x_2}(g^{sk})^2$$

Application: ZK Accumulator- the need for randomness

MANAGER

$Acc.v = g^{r(x_1+sk)(x_2+sk)}$

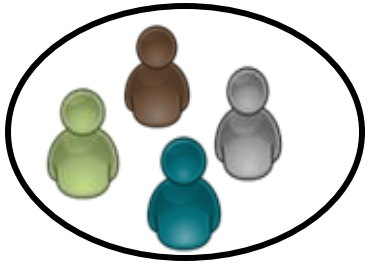
$S = \{x_1, x_2\}$

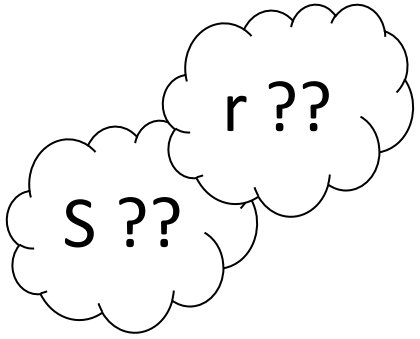
Add(x)

$Acc.v = g^{r\,\boldsymbol{r'}(x_1+sk)(x_2+sk)(\boldsymbol{x}+sk)}$

$S' = \{x_1, x_2, \boldsymbol{x}\}$

Set S (aux=r)





aux'={rr'}

Verification works!

Application: ZK Accumulator- Non-membership usually

Bilinear **ATSM09**, S=$\{x_i\}$, $x_i \in \mathbb{Z}_p$

$$\prod_{i=1}^{|S|}(x_i+sk) = c(sk)(y+sk)+d$$

Users (public parameters):
S=$\{x_i\}$, polynomial division
Manager (sk):
$\prod_{i=1}^{|S|}(x_i+sk) \in \mathbb{Z}$, used as exponent

$$g^u$$

$$u = \prod_{i=1}^{|S|}(x_i+sk) = c(sk)(y+sk)+d, \qquad d \neq 0$$

$$g^{ru}$$

Non-membership verification requires r

Application: ZK Accumulator-Non-membership by Ghosh et al

$$S \cap \{x\} \neq \emptyset$$

$$q_1[sk] \prod_{i=1}^{|S|}(x_i+sk) + q_2[sk]\,(y+sk) = 1$$

$$\overline{w_y} = (W_1, W_2) = (g^{(q_1[sk]+\gamma(y+sk))\,\boldsymbol{r^{-1}}}, g^{q_2[sk]-\gamma \prod_{i=1}^{|S|}(x_i+sk)})$$

$$e(W_1, \boldsymbol{Acc.v})e\left(W_2, g^x g^{sk}\right) = e(g,g)$$

Remove accumulator randomness

Application: ZK Accumulator- Non-membership by Ghosh et al

$$S \cap \{x\} \neq \emptyset$$

$$q_1[sk] \prod_{i=1}^{|S|}(x_i+sk) + q_2[sk]\,(y+sk) = 1$$

$$\overline{w_y} = (W_1, W_2) = (g^{(q_1[sk]+\gamma(y+sk))\,r^{-1}}, g^{q_2[sk]-\gamma \prod_{i=1}^{|S|}(x_i+sk)})$$

$$e(W_1, Acc.v)e\left(W_2, g^x g^{sk}\right) = e(g,g)$$

Add query/witness specific randomness

Application: ZK Accumulator- Non-membership by Ghosh et al

(+) r not needed for verification
(-) no witness update algorithm
Update → NonMemWitCreate: O(|S|)

$$\left(g^{r\prod_{i=1}^{|S|}(x_i+sk)}\right)$$

Add query/witness specific randomness

$$e(W_1, Acc.v)e\left(W_2, g^x g^{sk}\right) = e(g,g)$$

Application: ZK Accumulator-Modular Construction

(+) r not needed for verific
(-) no witness update algo
Update → NonMemWitCreate

$$e(W_1, Acc.v)e\left(W_2, g^x g^{sk}\right) = e(g,$$

Solution:
instantiate our generic modular universal construction with ZK accumulators with membership operations

Result:
Non-membership witness creation, Non-membership witness update: O(1)

Summary

In the secret key model:

1. We can hit optimal communication cost (*Positive Bilinear Accumulator with Optimal Communication Cost*)
2. We can have constant non-membership (*Universal Bilinear Accumulator with Constant Non-Membership Witness Creation*)
3. We can have constant ZK (*ZK Accumulator with Constant Non-Membership Witness Creation and Update*)

Available on ePrint soon